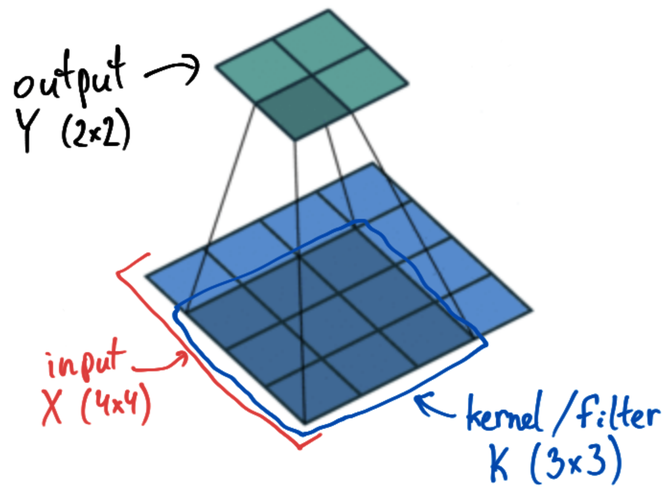
Convolutions

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} \quad K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$$

$$Y = X * K \quad \text{(convolution operator)}$$

$$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$



Vincent Dumoulin, Francesco Visin (2016) - A guide to convolution arithmetic for deep learning
https://github.com/vdumoulin/conv_arithmetic

$$y_{22} = x_{22}k_{11} + x_{23}k_{12} + x_{24}k_{13} + x_{32}k_{21} + x_{33}k_{22} + x_{34}k_{23} + x_{42}k_{31} + x_{43}k_{32} + x_{44}k_{33}$$

$$= \sum_{ij} \left[X_{2-4;2-4} \odot K \right]$$

$$y_{11} = \sum_{ij} \left[X_{1-3;1-3} \odot K \right]$$

↑ elementwise multiplication operator

Pooling

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$pool_{max,2\times2}(A) = \begin{bmatrix} \max\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\right) & \max\left(\begin{bmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{bmatrix}\right) \\ \max\left(\begin{bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}\right) & \max\left(\begin{bmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{bmatrix}\right) \end{bmatrix}$$

$$pool_{avg,2\times2}(A) = \begin{bmatrix} \overline{a_{11}} & \overline{a_{12}} \\ \overline{a_{21}} & \overline{a_{22}} \end{bmatrix}$$

Convolutions with matrix multiplication

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} \quad K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \quad Y = X * K$$

$$flatten(X) = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{21} & x_{22} & x_{23} & x_{24} & x_{31} & x_{32} & x_{33} & x_{34} & x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix}$$

$$K' = \begin{bmatrix} k_{11} & k_{12} & k_{13} & 0 & k_{21} & k_{22} & k_{23} & 0 & k_{31} & k_{32} & k_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{11} & k_{12} & k_{13} & 0 & k_{21} & k_{22} & k_{23} & 0 & k_{31} & k_{32} & k_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{11} & k_{12} & k_{13} & 0 & k_{21} & k_{22} & k_{23} & 0 & k_{31} & k_{32} & k_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{11} & k_{12} & k_{13} & 0 & k_{21} & k_{22} & k_{23} & 0 & k_{31} & k_{32} & k_{33} \end{bmatrix}$$

$$Y' = K' \cdot flatten(X) \qquad Y = reshape_{2\times2}(Y')$$

↑ matrix multiplication

Transposed convolutions

$$X \in \mathbb{R}^{4\times4} \xrightarrow{K} Y \in \mathbb{R}^{2\times2}$$

$$Y \in \mathbb{R}^{2\times2} \xrightarrow{K^T} X \in \mathbb{R}^{4\times4}$$

$$X' = K'^T \cdot flatten(Y) \qquad X = reshape_{4\times4}(X')$$

K'^T: 16×4, $flatten(Y)$: 4×1 → 16×1